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Note how your lesson objective ties to your state **Standards**

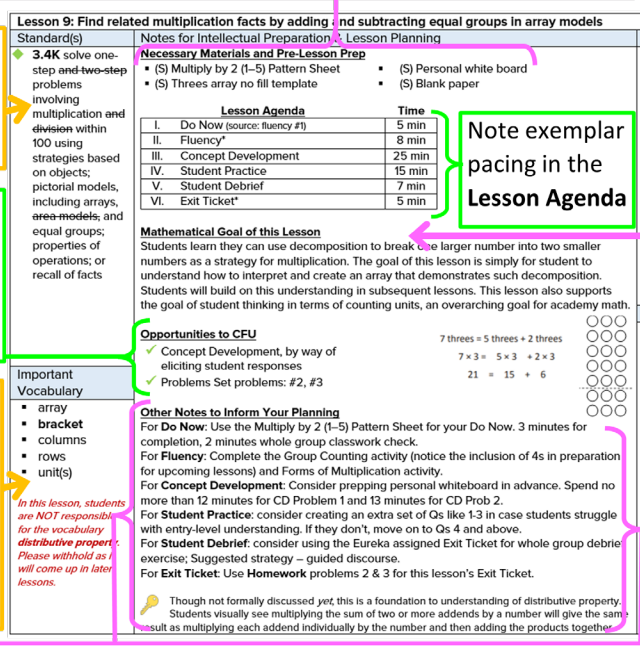
Note exemplar pacing in the **Lesson Agenda**

Use the **Mathematical Goal of the Lesson** to keep you focused on the appropriate student outcome

Plan purposeful questioning and responses using **Opportunities to CFU**

Plan instruction around what students need to Know & Do to be successful on the Exit Ticket using the identified **Student Know/Do Chart**

Plan to stress **Important Vocabulary** in the lesson. New vocab for the unit is indicated in bold



Find recommended lesson modifications, content knowledge boosters, and/or high-leverage instructional moves that may not be in your Teacher Edition located in **Other Notes to Inform Your Planning**

UNIT SYNOPSIS

This unit develops and strengthens students' understanding of functions and establishes the road map for the types of functions we will focus on throughout the course. The study of functions in Algebra 2 builds from the foundation developed in Algebra 1 when students learn to identify a function as a special type of relation and spend most of the course focused on linear and quadratic functions. Students will explore key features and attributes of the parent functions that will then become the focus of their own units later in this course. In this unit, students will strengthen their understandings of domain and range, develop ways to analyze key features of a variety of graphs, perform operations of functions including composition of functions, and explore the relationship between functions and their inverses if they exist.

Functions: A function is a relationship between variables in which each value of the input variable is associated with a unique value of the output variable. Functions can be represented in a variety of ways, such as with graphs, tables, equations, or words. Each representation can be particularly useful in certain situations. Families of functions consist of transformations of the simplest form of the function.

- A pairing of items from two sets constitutes a function if each item from one set pairs with exactly one item from the second set.
- Functions are grouped into families with specific qualities. Understanding these qualities and key attributes helps you graph and work with the various types of functions.
- You can add, subtract, multiply, and divide functions, and you can use one function as input for another function. It's important to consider the effect of these operations on the domain of the new function.
- The inverse of a function may or may not be a function. Given two functions, the composition of the functions can be used to determine if they are inverses of each other.

Equivalence: A single mathematical object may be represented by many different expressions. The facts about a mathematical object may be expressed by many different equations (or inequalities).

- A pairing of items from two sets constitutes a function if each item from one set pairs with exactly one item from the second set.
- You can add, subtract, multiply, and divide functions, and you can use one function as input for another function. It's important to consider the effect of these operations on the domain of the new function.

Properties: All the facts of arithmetic and algebra follow from certain properties or attributes.
Functions are grouped into families with specific qualities. Understanding these qualities and key attributes helps you graph and work with the various types of functions.

Misconceptions:

- *All functions have equations or are continuous*. Often students think that functions all can be described by equations, but often functions are discrete mappings from one finite set to another and do not represent continuous curves.
- *Domain is always x and range is always y*. Though we often default to x and y to represent relationships, these do not always appear as variables in a problem, especially in real world applications. It is more important that students understand that domain represents the possible inputs and range is the set of possible outputs. Similarly, some students may mistakenly think the domain is always "all real numbers."
- *All inverses of functions are functions.* Only functions that pass the Horizontal Line Test will have an inverse that is a function without restrictions on the domain.

Underdeveloped Concepts:

- Some students may only be able to create models of linear and quadratic functions at the start of Algebra 2 because these are the only functions they have primarily focused on in Algebra 1.
- Students may confuse inverse functions with inverse operations like addition and subtraction. It is important to emphasize that in inverse functions we switch the input and output variables, not the operations.
- Be very clear on the difference in meaning and notation between function multiplication and function compositions. Also be sure to demonstrate that composition of functions is not commutative.

Key Questions:

- What quantitative relationships in the real-world can functions be used to represent or approximate?
- How can function representations of real-world scenarios be used to make predictions or look for patterns?
- Why is the ability to represent quantitative relationships mathematically in a variety of ways important to solving problems in the real world?

CONTENT STANDARDS

Below are the standards addressed in this unit.

Readiness Standards	Supporting Standards
2A.2A Graph the functions $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = \lvert x \rvert$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.	**2A.2B** Graph and write the inverse of a function using notation such as $f^{-1}(x)$.
2A.2C Describe and analyze the relationship between a function and its inverse (quadratic and square root, logarithmic and exponential), including the restriction(s) on domain, which will restrict its range.	**2A.2D** Use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other.
2A.4C Determine the effect on the graph of $f(x) = \sqrt{x}$ when $f(x)$ is replaced by $af(x), f(x) + d, f(bx)$, and $f(x - c)$ for specific positive and negative values of a, b, c, and d.	**2A.7B** Add, subtract, and multiply polynomials.
2A.4F Solve quadratic and square root equations.	**2A.8A** Analyze data to select the appropriate model from among linear, quadratic, and exponential models.
2A.6E Solve absolute value linear equations.	

Focus on Disciplinary Literacy	
	Mathematical Process Standard (F) – Analyze mathematical relationships to connect and communicate mathematical ideas.
	Mathematical Process Standard (G) – Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

LEARNING SUPPORTS BY LESSON

There is a checkmark for the math support if the lesson	Lessons → Math Supports	L1	L2	L3	L4	L5	L6	L7	L8
makes a connection to prior content or from a previous unit or academic year	Access Prior Knowledge			✓	✓	✓	✓	✓	✓
uses familiar contexts or experiences to make the learning relevant to students	Real-World Connections	✓	✓	✓		✓	✓	✓	
makes use of graphic organizers	Graphic Organizers			✓			✓		
includes tools like rulers, protractors, patty paper, algebra tiles, etc.	Tools or Manipulatives								
incorporates tables, reference charts, displays, pictures, models, or color-coding	Visual Aids	✓	✓	✓	✓	✓	✓	✓	✓
includes definitions, examples vs. nonexamples, cognates, etc.	Vocabulary Supports	✓	✓	✓	✓	✓	✓	✓	✓
includes strategies that support language development									
asks students to discuss with their partner to prepare for whole class discussion	- Turn and Talk	✓	✓		✓		✓		
teacher facilitates a whole class discussion to debrief key learnings	- Guided Discussion	✓	✓	✓	✓	✓	✓	✓	✓
asks students to think independently, test their idea with a partner, and share whole group	- Think, Pair, Share	✓		✓	✓	✓	✓	✓	
includes sentence stems to support students with explanations	- Sentence Stems								
provides opportunities for students to work with a partner or a group	Peer Collaboration	✓	✓	✓	✓	✓	✓	✓	✓
uses mnemonics such as SohCahToa	Mnemonics								
includes websites or equipment that enhances the lesson	Technological Support	✓	✓	✓	✓				
content can be presented in different forms									
uses hands-on tools or manipulatives to represent the math	- Concrete								
uses drawings to represent the math	- Pictorial	✓	✓	✓	✓	✓	✓	✓	✓
uses numbers and number sentences to represent the math	- Abstract	✓	✓	✓	✓	✓	✓	✓	✓

The EFFL Model

Experience First, Formalize Later (EFFL) Model

Opening

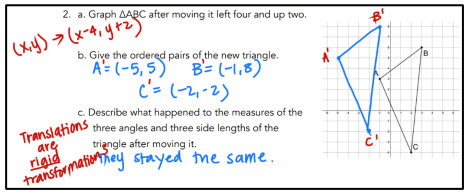
For every new lesson, the teacher begins by making the goals of the lesson crystal clear. The teacher does more than simply read the objective to the class. They make connections to previous learning, share how this learning fits into a bigger picture, or explain why this learning is important for future learning.

Activity / Interaction With New Material (INM)

For this part of the lesson, students work in pairs or groups of four to experience new content through an activity. Students might be discussing a proposed scenario, working with other groups, or doing a simulation. The student activity is designed for students to be able to do without the help of the teacher. Of course, the teacher is watching and listening in to conversations in order to formatively assess student understanding. The teacher provides questions, cues, and prompts (not answers) to help push groups forward when they are stuck or have made a mistake. As students begin to finish the activity, the teacher identifies students to write their work on the board. Most often, the teacher selects student work that will easily allow them to connect the experience to formal learning. Students write their work on the whiteboard in a single-color marker.

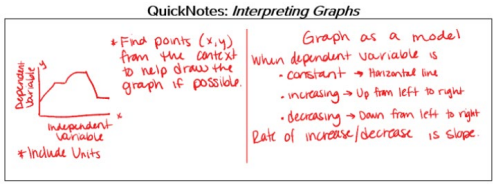
Debrief Activity

Once students have recorded their responses in their workbook (see blue writing to the right), the teacher calls the whole group back together for a debrief. It is in this discussion that the teacher will help students formalize the learning. The teacher connects the student activity experience to new vocabulary, definitions, formulas, and algorithms. The formal learning is attached specifically to the experiences of the activity so that students can enhance their constructed understanding of the new content. The teacher writes all of the formal learning in a different color in the margins of the activity **(see red writing to the right).** The students add these ideas in the margins on their activity page and often think of this as the formal "notes" of the lesson. In all of the answer keys we provide on Math Medic, the teacher formal learning points are provided in the margins in a **different color.**



QuickNotes

In this part of the lesson, the teacher uses the whole experience of the activity and the formalization in the debrief to summarize the learning from the lesson. Notice that we use the box to constrain the amount of formal "notes" that the teacher can provide.



Student Practice

Now that students have arrived at some new learning, they need to be able to apply it in new contexts. Most often we have students complete these questions in pairs and occasionally we select one question to use as an exit ticket. If we have time, we have students write solutions on the whiteboard.

Extra Practice

We typically give students around 3-5 "Extra Practice" problems for each lesson. We choose problems that are closely aligned with the Learning Objectives of the lesson. It is our belief that "less is more" here. We would rather students spend their Extra Practice time thinking deeply about just a few problems, rather than surface level thinking on many problems.

Slightly modified version of: https://www.calc-medic.com/post/experience-first-formalize-later#:~:text=%E2%80%9CExperience%20First%2C%20Formalize%20Later%E2%80%9D,at%20formal%20definitions%20and%20formulas.

Before You EFFL!

Here are helpful resources that you guide you in the right direction before your first EFFL lesson!

Why Should We EFFL?

The article advocates for the Experience First, Formalize Later (EFFL) teaching model, emphasizing its effectiveness in fostering deep understanding and flexible thinking in students. The author compares traditional teaching to a game of "Simon Says," where students merely mimic instructions without grasping underlying concepts. In contrast, EFFL encourages students to engage actively with problems, enhancing their ability to understand and apply calculus concepts creatively.

Tips for Lesson Planning

The article offers practical advice for effective lesson planning beyond the exhaustive and overly detailed approaches often emphasized during teacher training. It underscores the importance of thoughtful preparation but rejects the notion that teachers need to script every minute or detail of a class session.

Making the Most of Your EFFL Lesson Debrief

The article discusses the significance of the debriefing phase in the Experience First, Formalize Later (EFFL) lesson model, emphasizing its role in reinforcing learning and highlighting student contributions. The debrief session is seen as crucial for integrating academic vocabulary, emphasizing key lesson understandings, and valuing students' mathematical insights.

While You EFFL!

While each lesson may be unique in context and skills, all lessons benefit from the following practices:

Teacher Look Fors:

- ❑ Utilizing the Do Now to spark students' interest in the Activity.
- ❑ Use questioning to promote small group discussion and exploration, guided by monitoring questions.
- ❑ Connects Experience First to formal concepts using a colored pencil/pen to take notes along the margin during the Debrief.
- ❑ Facilitates whole-class discussions for students to reflect, share insights, and provides feedback that reinforces key concepts.
- ❑ Tracks time to adapt lesson pacing and support based on student response and engagement.

Students Look Fors:

- ❑ In the Activity, students engage in group work and discourse.
- ❑ Exploring the activity, testing hypotheses and approaches (trial & error).
- ❑ Take notes on key ideas and concepts using different colored pencil/pen to take notes along the margin.
- ❑ Share thoughts and ideas that demonstrate their approach to their work.

Other considerations

- During the **Experience First** phase, if most of your students seem stuck or disengaged, take a moment to pause, reset, and provide clear instructions. Some problems of the Activity are more suitable to do a whole-class discussion as a means to save some instructional time for Student Practice or the Exit Ticket. You are encouraged to adapt the EFFL (Experience First, Formalize Later) process to meet your students' needs while maintaining a focus on student-centered instruction.

ROADMAP

AT A GLANCE: Unit 1 – Foundations of Functions			
Day	Date	Lesson	Lesson Title
1		1	Using Multiple Strategies to Solve Equations
2		2	Interpreting Graphs
3		3	What is a Function?
4		4	Translating Functions
5			*Unit 1 Success Day 1 – Review topics from lessons 1.1 to 1.4*
6		5	Attributes of Functions
7		6	Combining Functions
8		7	Inverses of Relations and Functions
9		8	One-to-One Functions
10			*Unit 1 Success Day 2 – Use as needed based on your data*
11			*Unit 1 Success Day 3 – Unit Assessment Review*
12			End of Unit 1 Assessment

Date: _____

Lesson 1: Using Multiple Strategies to Solve Equations

Standard(s)

- **2A.4F** Solve quadratic and square root equations.
- **2A.6E** Solve absolute value linear equations.

Important Vocabulary

- equation
- solution
- variable
- **zeros**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- Graphing calculator/Desmos

Lesson Structure:

- Do Now (8 min)
- INM (20 min)
- Debrief (7 min)
- Student Practice (15 min)
- Exit Ticket (10 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to use graphs, tables, and algebraic methods to find solutions to linear, quadratic, and absolute value equations or to approximate a solution to an equation in context.

Opportunities to CFU

✓ INM #2-6

✓ Student Practice #1

Other Notes to Inform Your Planning

We're taking today to work on solving equations. Students should know how to solve all of the equations presented. However, we know as educators that students come into our classes with different prior experiences. We don't want to assume they're fluent at something this foundational now when this is a skill they MUST have in order to get to more advanced content later on. This lesson will also help you to get an understanding of how comfortable your students are with solving nonlinear equations.

The application problems in the Student Practice allow students to select the method of their choosing to best find a solution to the problem. Allow students to select any method that is mathematically sound in these problems.

Lesson Look Fors

Look for teachers to...

- ❑ Guide student conversations but let students do the conceptual lift.
- ❑ Build on students' prior knowledge of how to solve equations from Algebra 1 and Geometry.

Look for students to...

- ❑ Connect points of intersection of graphs to solutions when y is a specific value.
- ❑ Solve equations using graphs, tables, or algebraic methods.

Student Know/Do Chart

Students can

Do — Solve equations algebraically by isolating the variable and using inverse operations.

Do — Solve equations by graphing by looking for the point of intersection between the curve and the horizontal line of a constant value or by splitting an equation into two equations.

Know — Know that the solution to an equation is the value or values that make the equation a true statement when input for the variable.

Date: _____

Lesson 2: Interpreting Graphs

Standard(s)

◆ **2A.8A** Analyze data to select the appropriate model from among linear, quadratic, and exponential models.

Important Vocabulary

- **continuous**
- **dependent variable**
- **discontinuous**
- domain
- function
- **independent variable**
- range
- relation
- **x-intercept**
- **y-intercept**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- Computer for Desmos Activity

Lesson Structure:

- Do Now (5 min)
- INM (20 min)
- Debrief (7 min)
- Student Practice (20 min)
- Exit Ticket (8 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to identify the independent and dependent variables for a model. Students will create graphs to model situations and use Desmos to interact with real-world scenarios and connect them to data points.

Opportunities to CFU

- ✓ INM #3 & 4
- ✓ Debrief
- ✓ Student Practice #1-4

Other Notes to Inform Your Planning

We're going to be using Desmos for the next 3 lessons. Each has a Desmos Classroom Activity to go along with the lesson. You can find all of Desmos activities we will be using in the Math Medic Desmos Activities Collection (the names match the Math Medic titles). Today we will be using the Desmos Classroom Activity called Graphing Stories. If you haven't used Desmos Classroom Activities yet, don't worry! They're very user friendly and they have an excellent set of tutorials on their resources tab. To use this activity with your students, you can create a single session code and share it with your students. You can fill this in on the lesson activity page for students. We've edited the original Desmos activity to follow along with our lesson. If you want to make further edits you can save a copy and edit it.

Once you get students all set up with Desmos, they can get working. We'd recommend working in pairs so that it is easier to view the screen. Make sure students are recording their work on their lesson and not just typing responses into Desmos.

Lesson Look Fors

Look for teachers to...

- ❑ Set clear expectations for use of Desmos Activities for the first time. Clear procedures will prevent class management issues.
- ❑ Develop the connection between the domain and range beyond x and y.

Look for students to...

- ❑ Model scenarios by sketching graphs as well as selecting sketches to match a given description.
- ❑ Identify appropriate independent and dependent variables for given scenarios.

Student Know/Do Chart

Students can

Do Create graphs to model situations.

Do Identify the independent and dependent variables for a model.

Know Know a function can be created to model real world scenarios if an independent and dependent variable can be defined.

Date: _____

Lesson 3: What is a Function?

Standard(s)

- **2A.2** The student applies mathematical processes to understand that functions have distinct key attributes and understand the relationship between a function and its inverse.

Important Vocabulary

- relation
- function
- **one-to-one function**
- domain
- range
- **vertical line test**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- Computer for Desmos Activity

Lesson Structure:

- Do Now (5 min)
- INM (20 min)
- Debrief (8 min)
- Student Practice (20 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to identify the domain and range of a relation and determine if a relation is a function. Students will use function notation when writing and evaluating functions.

Opportunities to CFU

✓ After INM

✓ Student Practice #1 & 3

Focus on Disciplinary Literacy

INM #7 & Student Practice

Other Notes to Inform Your Planning

In the last lesson, we used a Desmos activity where students watched videos showing different scenarios and predicted what the graphs would look like. Today we will focus on just the first video (the slide). Students will use the video to create a function for waist height and time. They'll represent this function verbally, numerically, and graphically.

Here is the Desmos activity for today's lesson. We edited the Desmos activity that we used in the last lesson to only show the first video. This is really all students need to complete the lesson. To create the table of important values, students should rewatch and pause the video as many times as needed. If groups get slightly different values here, that's fine.

Lesson Look Fors

Look for teachers to…

- ❑ Guide student to the Desmos Activity but let students do the conceptual lift.
- ❑ Connect students' understanding of inputs and outputs to the proper use of function notation.

Look for students to…

- ❑ Correctly identify the domain and range of a relation.
- ❑ Determine if a relation is a function. If it is, properly use function notation.

Student Know/Do Chart

Students can

Do — Properly evaluate a function for a real input or a variable given an equation or a graph.

Do — Determine if a relation is a function.

Know — Function notation is only used for functions, so if the relation is given as equal to f(x) then it is a function.

Date: _____

Lesson 4: Translating Functions

Standard(s)

- **2A.2A** Graph the functions $f(x) = \sqrt{x}$, ~~$f(x) = 1/x$~~, ~~$f(x) = x^3$~~, ~~$f(x) = \sqrt[3]{x}$~~, ~~$f(x) = b^x$~~, ~~$f(x) = |x|$~~, and ~~$f(x) = \log_b(x)$~~ where ~~b~~ is 2, 10, and ~~e~~, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.
- **2A.4C** Determine the effect on the graph of $f(x) = \sqrt{x}$ when f(x) is replaced by af(x), f(x) + d, f(bx), and f(x - c) for specific positive and negative values of a, b, c, and d.

Important Vocabulary

- function
- **rigid transformation**
- **horizontal shift**
- **vertical shift**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- Computer for Desmos Activity

Lesson Structure:

- Do Now (5 min)
- INM (20 min)
- Debrief (5 min)
- Student Practice (15 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to translate the graph of a function using $y = k + f(x - h)$. Students will write and simplify the translated equation of a function.

Opportunities to CFU

✓ INM #1, 2, & 5

✓ Student Practice #1 & 2

Other Notes to Inform Your Planning

This lesson has a Desmos Classroom Activity for students to use to investigate how to **translate** functions. Create a single session code and write it in for the students or assign the activity to your classes. Unfortunately, this Desmos activity can't be copied and edited so you'll have to use this version as is.

Focus on Disciplinary Literacy

INM & Debrief

Give students about 15 minutes to play around with moving the graphs and matching them up. This is a great opportunity for students to practice writing translations with function notation which they often find confusing.

Our goal is that by the end of the activity, students can describe how $y = f(x + h) + k$ has been shifted from $f(x)$.

If groups finish the lesson early, they can continue working on the later slides in the Desmos activity. This lesson is a little shorter than normal, so you've got some time.

Lesson Look Fors

Look for teachers to…

- ❑ Guide student to the Desmos Activity but let students do the conceptual lift.
- ❑ Discuss equivalent forms of functions and how the translation may look different when equations are simplified.

Look for students to…

- ❑ Translate the graph of functions horizontally and vertically.
- ❑ Describe what translations would happen for a given function without sketching the curve.

Student Know/Do Chart

Students can

Do: Translate the graph of a function using $y = k + f(x - h)$.

Know: Translating a shape horizontally or vertically does not change the shape.

Know: To translate the graph of a function using $y = k + f(x - h)$, the function moves up or down k units and left or right h units.

Date: _____

Lesson 5: Attributes of Functions

Standard(s)

2A.2A graph the functions $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.

Important Vocabulary

- **asymptotic behavior**
- horizontal asymptote
- **maximum (absolute and relative)**
- **minimum (absolute and relative)**
- **reflectional symmetry**
- **rotational symmetry**
- **symmetry**
- vertical asymptote
- x-intercept
- y-intercept
- zeros

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbooks

Lesson Structure:

- Do Now (5 min)
- INM (27 min)
- Debrief (5 min)
- Student Practice (18 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to identify the intercepts, symmetry, asymptotes, and the extrema of a function. Students will find the x and y intercepts of given functions and will identify if a graph of a function is symmetric about an axis or the Origin. Students will identify when there is an asymptote and how asymptotic behavior affects the function. Students will also identify the relative maximum and minimum.

Opportunities to CFU

- ✓ Each category of the INM
- ✓ Debrief
- ✓ Student Practice

Focus on Disciplinary Literacy

Debrief

Other Notes to Inform Your Planning

In this lesson the students will continue learning about the attributes of functions: intercepts and symmetry. The lesson will use parent functions to introduce the procedure for identifying intercepts and symmetry, different types of symmetry will be deepened. Students will also look at what parent functions have asymptotes and will reinforce the process to find the domain of a function.

Students should have a basic idea of how parent functions are represented and how to determine their domain, range, symmetry, and intercepts.

Lesson Look Fors

Look for teachers to…

- ❑ Connect symmetry to real world applications.
- ❑ Support students in substituting zero and solving for the remaining variable. Some students may still struggle with solving equations.
- ❑ Work with students to comprehend asymptotic behavior and relative extrema as some students may struggle with this at first. Zooming in on graphs may help.

Look for students to…

- ❑ Set a function equal to zero and solve to find the x-intercepts (zeros).
- ❑ Input zero for x into a function and solve to find the y-intercepts.
- ❑ Analyze the graphs of a variety of functions to identify the behavior of the curve near asymptotes and at relative extrema.

Student Know/Do Chart

Students can

- Do: Find the x and y intercepts of a given function from a graph or equation.
- Do: Identify relative and absolute maximums and minimums of a given curve.
- Do: Find the vertical and horizontal asymptotes of a rational function.
- Know: Know the types of symmetry that can be present in a function.
- Know: Know the key parent functions and the attributes of their graphs.

Date: _____

Lesson 6: Combining Functions

Standard(s)

- **2A.2D** Use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other.
- **2A.7B** Add, subtract, and multiply polynomials.

Important Vocabulary

- **composition of functions**
- **input function**
- domain

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook

Lesson Structure:

- Do Now (6 min)
- INM (25 min)
- Debrief (7 min)
- Student Practice (15 min)
- Exit Ticket (7 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to combine functions and constants to create new functions using addition, subtraction, multiplication, and division. Students will evaluate combinations of functions for given input values. When students are given two functions, they will compose new functions by inputting one into the other. Students will also evaluate a composition of functions for given input values.

Opportunities to CFU

- ✓ INM #1, 2, & 7
- ✓ Debrief

Focus on Disciplinary Literacy

Debrief

Other Notes to Inform Your Planning

In today's lesson we are going to start with the idea of combining functions through addition, subtraction, multiplication and division. Revenue, expenses and profit lend themselves really nicely to this. Students should be comfortable calculating these but they may be unfamiliar with the vocabulary. Have students work thought the entire activity with their groups. The later problems build students to composition of functions.

Lesson Look Fors

Look for teachers to…

- ❑ Use the guiding questions from the TE.
- ❑ Support students in understanding the connection between revenue, expenses, and profit.
- ❑ Explain how students can do operations with functions like subtraction and division to create new functions like the profit function.

Look for students to…

- ❑ Work in groups or pairs and combine the correct equations using the proper operations for the given scenario.
- ❑ Use a variety of tables, equations, and function notation to describe and perform function operations including composition of functions.

Student Know/Do Chart

Students can

Do: Combine functions and constants to create new functions using addition, subtraction, multiplication, and division.

Do: Compose two functions and evaluate a composition of functions for given input values.

Know: How to perform the four basic operations using functions and how to compose two functions when given in function notation, for example $f(g(x))$.

Date: _____

Lesson 7: Inverses of Relations and Functions

Standard(s)

- **2A.2C** Describe and analyze the relationship between a function and its inverse (quadratic and square root, ~~logarithmic and exponential~~), including the restriction(s) on domain, which will restrict its range.
- **2A.2B** Graph and write the inverse of a function using notation such as $f^{-1}(x)$.
- **A2.2D** Use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other.

Important Vocabulary

- **composition of functions**
- domain
- **inverse function**
- range

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Debrief (7 min)
- Student Practice (18 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to write the inverse of a given function and determine if two functions are inverses algebraically using composition of functions. Students will graph the inverse of a function and determine the domain and range of an inverse function and describe its relationship to the original function.

Opportunities to CFU

- ✓ INM #4, 5, 8, & 9
- ✓ Student Practice #1-4

Focus on Disciplinary Literacy

INM #8 & 9 and Debrief

Other Notes to Inform Your Planning

In today's lesson we are introducing the idea of an inverse function. To find the inverse algebraically, we switch the input and output and then undo it with inverse operation. When you see this in a context, the whole idea of switching x and y and then solving for y clearly doesn't work.

MathMedic was careful with this lesson to make sure everything we're teaching students is mathematically sound without being overcomplicated. Something that helps a lot with this is teaching inverses in context. This makes it much easier to talk about inputs and outputs without saying x and y. When teaching this lesson, we want to keep bringing it back to inputs and outputs. Ask students, "What is going into that function? What is coming out of that function?" This makes it much easier for a student to understand why an inverse undoes the function.

The second half of the lesson focuses on graphs of inverse functions. We want students to be thinking about the idea that when we have a function and its inverse the inputs and outputs are switching. What this means is that the graphs of the function and its inverse are going to be reflections over the line y=x.

Lesson Look Fors

Look for teachers to...

- ❑ Use the guiding questions from the TE.
- ❑ Support students as they verify inverses using compositions and as they reflect graphs across the line $y = x$. Both tasks can challenge some students.

Look for students to...

- ❑ Recognize that since the input and output switch with an inverse, that means the axes will also switch.
- ❑ Compose functions to verify they are inverses.
- ❑ Find the domain and range of inverse functions.

Student Know/Do Chart

Students can

- Do: Write the inverse of a given function and determine if two functions are inverses algebraically using composition of functions.
- Do: Graph the inverse of a function and determine the domain and range of the inverse.
- Know: The graphs of the function and its inverse are reflections over the line $y = x$.
- Know: The domain and range of a function switch for the inverse function.

Date: _____

Lesson 8: One-to-One Functions

Standard(s)

- **A2.2C** Describe and analyze the relationship between a function and its inverse (quadratic and square root, ~~logarithmic and exponential~~), including the restriction(s) on domain, which will restrict its range.
- **A2.2B** Graph and write the inverse of a function using notation such as $f^{-1}(x)$.
- **A2.2D** Use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other.

Important Vocabulary

- **horizontal line test**
- one-to-one function
- vertical line test

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator

Lesson Structure:

- Do Now (5 min)
- INM (20 min)
- Debrief (5 min)
- Student Practice (25 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to determine whether a function is one-to-one. Students will use compositions of functions to verify if two functions are inverses. Students will also use the Horizontal Line Test to test if the inverse of a function will also be a function. Students will also analyze what restriction to the domain may be necessary to make a function be one-to-one.

Opportunities to CFU

- ✓ After INM #5
- ✓ End of INM
- ✓ Debrief

Focus on Disciplinary Literacy

INM #5 & 13 and Debrief

Other Notes to Inform Your Planning

Throughout this lesson it is important that students remember previous concepts and strategies developed during Unit 1. Students will determine the inverse of a function graphically and if a function is one-to-one by using a strategy similar to the Vertical Line Test, called the Horizontal Line Test.

Lesson Look Fors

Look for teachers to…

- ❑ Support students that are still developing their understanding of how to find the inverse of a function.
- ❑ Clarify how and when to use the Horizontal Line Test (on the original function before finding the inverse).

Look for students to…

- ❑ Utilize the Vertical and Horizontal Line Tests to determine if a graph of a curve is a function and if the inverse would be a function.

Student Know/Do Chart

Students can

Do: Determine if a given function is one-to-one and find it's inverse function if it is.

Know: The Horizontal Line Test states that if no horizontal line can be drawn that would intersect the graph of a function in more than one point, then the function is one-to-one.

Recommended Unit 1 Success Days Material and Resources

Date: _____

To review topics taught up to 1.4, use the following resources. Your exit ticket data should be used to determine individualized needs. The resources can be used in small groups, whole groups, or independent groups and be integrated with other classroom routines, like computer aligned practice and teacher-led groups.

To review or practice relations & functions, use...

Sample Activities & Tasks:

- Relations and Functions Practice (answers)
- Creating a Function Story

Content Video Lessons:

- Domain and Range of Relations and Functions
- Determining Whether Relations are Functions
- Using the Vertical Line Test

To review or practice using functions notation and evaluating functions, use...

Sample Activities & Tasks:

- Name That Notable Functionist (Function Notation Skill-Builder)
- Function Notation Practice

Content Video Lessons:

- Evaluate Functions
- Writing Functions Using Function Notation

To review other topics taught in Unit 1, use the following resources. Your exit ticket data should be used to determine individualized needs. The resources can be used in small groups, whole groups, or independent groups and be integrated with other classroom routines, like computer aligned practice and teacher-led groups.

Sample Activities & Tasks:

- Domain and Range Desmos Card Sort
- Function Attributes Exercises
- Intercepts Desmos Activity
- Function Attributes Exercises
- Discovering Asymptotes Desmos Activity
- Operations on Functions Exercises
- Composing Functions Desmos Exploration

Content Video Lessons:

- Domain and Range of Relations and Functions
- Worked Example: Domain and Range from a Graph
- Worked Example: Determining Domain Word Problems
- Introduction to Intercepts
- *x*-Intercept of a Line
- Intercepts from an Equation
- Intercepts from a Table
- Introduction to Function Symmetry
- Graphical Symmetry of Functions
- What are Asymptotes?
- Introduction to Minimum and Maximum Points
- Worked Example: Absolute and Relative Extrema
- Local and Absolute Extrema
- Adding Functions
- Subtracting Functions
- Multiplying Functions
- Quotient of Functions
- Intro to Function Composition
- Evaluating Composite Functions
- Domain of a Composition Part 1
- Domain of a Composition Part 2
- Domain of a Composition Common Mistake

Date: _____

Unit 1 Exam

Standard(s)	Notes for Intellectual Preparation & Lesson Planning
◆ **2A.2A** Graph the functions $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = \lvert x \rvert$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval. **Problems #2,4,6, 11abc** ◆ **2A.2C** Describe and analyze the relationship between a function and its inverse (quadratic and square root, logarithmic and exponential), including the restriction(s) on domain, which will restrict its range. **Problems #9** ◆ **2A.4C** Determine the effect on the graph of $f(x) = \sqrt{x}$ when $f(x)$ is replaced by $af(x), f(x) + d, f(bx)$, and $f(x - c)$ for specific positive and negative values of a, b, c, and d. **Problems #5** ◆ **2A.2B** Graph and write the inverse of a function using notation such as $f^{-1}(x)$. **Problems #9** ◆ **2A.2D** Use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other. **Problems #8,10** ◆ **2A.7B** Add, subtract, and multiply polynomials. **Problems #7** ◆ **2A.8A** Analyze data to select the appropriate model from among linear, quadratic, and exponential models. **Problems #3**	**Necessary Materials and Pre-Lesson Prep** ▪ Algebra 2 Unit 1 Exam ▪ Assessment Companion for Algebra 2 Unit 1 Exam found on Curriculum Corner **Notes to Inform Your Planning** Review the Unit 1 Exam on Curriculum Corner. Internalize & create an exemplar for the assessment prior to teaching the unit as part of unpacking the unit. Use the exemplar to spar with the answer key provided on Curriculum Corner. Administer the Unit 1 Exam following guidance on the Scope & Sequence to ensure the scoring deadline is met.

UNPACKED STANDARDS

Focus standards for this unit.

Standards Clarification		
Standards	**Specificity**	**Notes/Explanations/Examples**
2A.2A graph the functions $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = \lvert x \rvert$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval	Including, but not limited to: • Representations of functions, including graphs, tables, and algebraic generalizations ○ Square root, $f(x) = \sqrt{x}$ ○ Rational (reciprocal of x), $f(x) = \frac{1}{x}$ ○ Cubic, $f(x) = x^3$ ○ Cube root, $f(x) = \sqrt[3]{x}$ ○ Exponential, $f(x) = b^x$, where b is 2, 10, and e ○ Absolute value, $f(x) = \lvert x \rvert$ ○ Logarithmic, $f(x) = \log_b(x)$, where b is 2, 10, and e • Connections between representations of families of functions • Comparison of similarities and differences of families of functions • Domain and range of the function ○ Domain – set of input values for the independent variable over which the function is defined ○ Range – set of output values for the dependent variable over which the function is defined ○ Representation for domain and range ○ Domain and range of the function versus domain and range of the contextual situation • Key attributes of functions ○ Intercepts/Zeros ○ Symmetries ○ Asymptotic behavior – behavior such that as x approaches infinity, $f(x)$ approaches a given value ○ Use key attributes to recognize and sketch graphs ○ Application of key attributes to real-world problem situations	• *Grade Level(s):* ○ *The notation $\Re$ represents the set of real numbers, and the notation Z represents the set of integers.* ○ *Algebra I studied parent functions f(x) = x, f(x) = x^2, and f(x) = b^x and their key attributes.* ○ *Precalculus will study polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions.* ○ *Various mathematical process standards will be applied to this student expectation as appropriate.*
2A.2C describe and analyze the relationship between a function and its inverse (quadratic and square root, logarithmic and exponential), including the restriction(s) on domain, which will restrict its range	Including, but not limited to: • Relationships between functions and their inverses ○ All inverses of functions are relations. ○ Inverses of one-to-one functions are functions.	• *Grade Level(s):* ○ *Algebra I determined if relations represented a function.*

Standards Clarification		
Standards	**Specificity**	**Notes/Explanations/Examples**
	○ Inverses of functions that are not one-to-one can be made functions by restricting the domain of the original function, $f(x)$. • Characteristics of inverse relations ○ Interchange of independent (x) and dependent (y) coordinates in ordered pairs ○ Reflection over $y = x$ • Domain and range of the function versus domain and range of the inverse of the given function • Functionality of the inverse of the given function tabularly and graphically ○ Linear function and linear function, $f(x) = mx + b$ ○ Quadratic function and square root function, $f(x) = x^2$ and $f(x) = \sqrt{x}$ ○ Cubic function and cube root function, $f(x) = x^3$ and $g(x) = \sqrt[3]{x}$ ○ Exponential function and logarithmic function, $f(x) = b^x$ and $g(x) = \log_b(x)$ where b is 2, 10, and e	○ *Algebra II introduces inverse of a function and restricting domain to maintain functionality.* ○ *Various mathematical process standards will be applied to this student expectation as appropriate.*
2A.2B graph and write the inverse of a function using notation such as $f^{-1}(x)$	Including, but not limited to: • Inverse of a function – function that undoes the original function. When composed $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. • Inverses of functions on graphs ○ Symmetric to $y = x$ • Inverses of functions in tables ○ Interchange independent (x) and dependent (y) coordinates in ordered pairs • Inverses of functions in equation notation ○ Interchange independent (x) and dependent (y) variables in the equation, then solve for y • Inverses of functions in function notation ○ $f^{-1}(x)$ represents the inverse of the function $f(x)$.	• *Grade Level(s):* ○ *Algebra II introduces inverse of a function.* ○ *Various mathematical process standards will be applied to this student expectation as appropriate.*
2A.2D use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other	Including, but not limited to: • Composition of functions – process of substituting one function into another function to create a new function such that the range of one function becomes the domain of the other function • Composition notation given $f(x)$ and $g(x)$ ○ Verbal ○ Symbolic • Characteristics of inverse relations	• *Grade Level(s):* ○ *Algebra II introduces inverse of a function.* ○ *Various mathematical process standards will be applied to this student expectation as appropriate.*

Standards Clarification		
Standards	Specificity	Notes/Explanations/Examples
	○ Interchange of independent (x) and dependent (*y*) coordinates in ordered pairs ○ Interchange of independent (x) and dependent (*y*) coordinates in an equation and resolving for *y* ○ Reflection over the $f(x) = x$ line ○ Domain of the function becomes an appropriate range of the inverse function ○ Range of the function becomes an appropriate domain of the inverse function ○ Composed as $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$ • Domain and range of the function versus domain and range of the inverse of the given function	

VERTICAL STANDARDS

This section details the **progression** of key student expectations in the courses **before** and **after** this course. This will help you understand what **prior knowledge skills to build upon** and guide you in knowing what **skills you are preparing your students** for in the subsequent course.

Algebra 1	Algebra 2	Pre-Calculus
• **A.2A** Determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities. • **A.3C** Graph linear functions on the coordinate plan and identify key features, including the x-intercept, y-intercept, zeros, and slope in mathematical and real-world problems. • **A.6A** Determine the domain and range of quadratic functions and represent the domain and range using inequalities. • **A.10A** Add and subtract polynomials of degree one and degree two. • **A.10B** Multiply polynomials of degree one and degree two. • **A.10C** Determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend. • **A.12A** Decide whether relations represented verbally, tabularly, graphically, and symbolically define a function. • **A.12B** Evaluate functions, expressed in function notation, given one or more elements in their domains.	• **2A.2A** Graph the functions $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = \lvert x \rvert$, and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval. • **2A.2B** Graph and write the inverse of a function using notation such as $f^{-1}(x)$. • **2A.2C** Describe and analyze the relationship between a function and its inverse (quadratic and square root, logarithmic and exponential), including the restriction(s) on domain, which will restrict its range. • **2A.2D** Use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other. • **2A.4C** Determine the effect on the graph of f(x)= √x when f(x) is replaced by af(x),f(x) + d,f(bx), and f(x - c) for specific positive and negative values of a, b, c, and d. • **2A.4F** Solve quadratic and square root equations. • **2A.6E** Solve absolute value linear equations. • **2A.7B** Add, subtract, and multiply polynomials. • **2A.8A** Analyze data to select the appropriate model from among linear, quadratic, and exponential models.	• **(2.A)** Use the composition of two functions to model and solve real-world problems. • **(2.B)** Demonstrate that function composition is not always commutative. • **(2.C)** Represent a given function as a composite function of two or more functions. • **(2.D)** Describe symmetry of graphs of even and odd functions. • **(2.E)** Determine an inverse function, when it exists, for a given function over its domain or a subset of its domain and represent the inverse using multiple representations. • **(2.F)** Graph exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions. • **(2.G)** Graph functions, including exponential, logarithmic, sine, cosine, rational, polynomial, and power functions and their transformations, including $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d, in mathematical and real-world problems. • **(2.I)** Determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum, relative minimum, zeros, asymptotes, and intervals over which the function is increasing or decreasing.